Spin correlations in the reaction $\pi^{\pm}\vec{D} \to \vec{\Sigma}^{\pm}\Theta^{+}$ and the parity of Θ^{+}

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We analyze two types of spin observables in the reaction $\pi\vec{D} \to \vec{\Sigma}\Theta^{+}$ near the threshold. One concerns the spin-transfer coefficients K_x^x and K_z^z. The second one is the deuteron spin anisotropy. These observables are sensitive to the Θ^{+} parity and can be used as a tool for the Θ^{+} parity determination.

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I. INTRODUCTION

The first evidence for the pentaquark particle Θ^{+} discovered by the LEPS collaboration at SPring-8 [1] was subsequently confirmed in other experiments [2]. However some other experiments fail to find the Θ^{+} signal (for a review see [3]). None of the experiments with positive signal of Θ^{+} determined the spin and the parity of the Θ^{+}. Since most of evidences of the pentaquark came from photoproduction, then naturally there were several proposals for determination of the parity of Θ^{+}, π_Θ, from photoproduction processes using single and double polarization observables. However, these suggestions are based on model-dependent calculations. Only triple spin observables may be considered as reliable candidates for a determination of π_Θ, but the practical implementation of this method is rather hard (for a review see [4]). Therefore, one should look for other reactions and observables for elucidating π_Θ, since the determination of π_Θ is challenging to pin down the nature of Θ^{+}

In Ref. [5], Thomas, Hicks and Hosaka have proposed an attractive method to unambiguously determine the Θ^{+} parity in the reaction $\vec{p}\vec{p} \to \Sigma^{+}\Theta^{+}$ close to the production threshold. They have shown that the initial proton-proton spin state with spin $S = 0\,(1)$ is completely defined by the the Θ^{+} parity ($\pi_\Theta = +(-)$). The method is solely based on the Pauli exclusion principle and the total spin and parity conservation as well, and therefore is model independent, indeed. This idea with utilizing other double spin observables (spin-spin correlations) in $NN \to Y\Theta^{+}$ reactions has been further developed by Hanhart et al. [6, 7], Rekalo and Tomasi-Gustafsson [8, 9], Uzikov [10, 11], and Nam, Hosaka and Kim [12]. In particular, the focus was put on an analysis of the spin-transfer coefficients which are sensitive to the production mechanism and Θ^{+} parity. It seems to be interesting and important to elaborate further and alternative methods for an unambiguous determination of π_Θ which serve an independent check of internal Θ^{+} properties.

In our Communication we consider the reaction $\pi^{\pm}\vec{D} \to \vec{\Sigma}^{\pm}\Theta^{+}$ near the threshold. We analyze (i) the spin-transfer coefficients $K_i^i\,(i = x, y, z)$, where the spin is transferred from the polarized deuteron to the outgoing Σ, and (ii) the deuteron spin anisotropy $\mathcal{A}$, which defines the production cross section as a function of the angle between the deuteron spin and direction of the pion beam. The latter observable has an obvious advantage because it bases on single polarization (polarized deuteron target) measurements.

For the sake of clarity, in the following we limit our discussion for determining the Θ^{+} parity to an isoscalar spin-1/2 Θ^{+}. In fact, most theories predict J^P of Θ^{+} to be $1/2^{+}$ or $1/2^{-}$. The generalization for higher spin of Θ^{+} is straightforward. Our consideration resembles the previous works [6, 10] on NN reactions. The main difference is that in our case the total isospin and the spin of the np system (deuteron) in the initial state are fixed. The deuteron polarization with respect to the beam direction provides additional observables compared to the NN reaction.

II. SPIN-TRANSFER COEFFICIENTS

Spin-transfer coefficients for the reaction $\pi\vec{D} \to \vec{\Sigma}\Theta^{+}$[1] are related to the production amplitude T as [13]

$$K_i^f = \frac{\mathrm{Tr}[T\,S_i(D)T^{\dagger}\sigma_f(\Sigma)]}{\mathrm{Tr}[T\,T^{\dagger}]}\ , \tag{1}$$

where σ_f and S_i are the Pauli spin-$\frac{1}{2}$ and spin-1 spin matrices, respectively. The latter ones are defined as

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ S_y = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$
$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\ . \tag{2}$$

[1] In what follows, the charge indicating superscript ($\pm$) will be suppressed for $\pi^{\pm}$ and $\Sigma^{\pm}$.

In the near-threshold region we assume that the final states with the orbital momentum $L_f > 0$ are suppressed and, therefore, the production amplitude has the following form

$$T_{m,m';M_D} = \sum_{JM,LM_L} \langle \frac{1}{2} m \frac{1}{2} m' | JM \rangle \langle 1 M_D L M_L | JM \rangle \times Y_{LM_L}(\hat{\mathbf{k}}) a_L^J \ , \quad (3)$$

where $\hat{\mathbf{k}}$ is direction of the pion beam momentum with respect to the quantization axis, m and m' are the spin projections of Σ and Θ^+, respectively, L is the orbital momentum in the initial state, J stands for the total angular momentum, and a_L^J denotes the partial amplitude.

For positive π_Θ the orbital momentum in the initial state must be $L = 1$ which results in $J = 0, 1$. The spin-transfer coefficients for the case when the quantization axis is taken along the beam direction read

$$K_z^z = \frac{3r^2}{1+3r^2} \ , \quad (4a)$$

$$K_x^x = K_y^y = \frac{\sqrt{6}\alpha r}{1+3r^2} \quad (4b)$$

with

$$a_1^0 = |a_1^0| \, e^{i\phi_1^0}, \ a_1^1 = |a_1^1| \, e^{i\phi_1^1}, \ r = |a_1^1/a_1^0|,$$
$$\alpha = \cos\delta^+ \equiv \cos(\phi_1^0 - \phi_1^1) \ .$$

Taking the ratio r from Eq. (4a),

$$r = \sqrt{\frac{K_z^z}{3(1-K_z^z)}} \ , \quad (5)$$

one can express K_x^x through K_z^z as

$$K_x^x = \pm\alpha\sqrt{2K_z^z(1-K_z^z)}, \quad (6)$$

and one finds the constraints for K_i^i

$$0 \le K_z^z \le 1 \ , -|\alpha|\frac{\sqrt{2}}{2} \le K_x^x \le |\alpha|\frac{\sqrt{2}}{2}. \quad (7)$$

For negative π_Θ the orbital momentum in the initial state must be $L = 0, 2$ and the total angular momentum is $J = 1$. The spin-transfer coefficients are expressed through the partial amplitudes

$$K_z^z = \frac{2 + 2\sqrt{2}\beta q + q^2}{3(1+q^2)} \ , \quad (8a)$$

$$K_x^x = K_y^y = \frac{2 - \sqrt{2}\beta q - 2q^2}{3(1+q^2)} \ , \quad (8b)$$

$$(8c)$$

with

$$a_0^1 = |a_0^1| \, e^{i\phi_0^1}, \ a_2^1 = |a_2^1| \, e^{i\phi_2^1}, \ q = |a_2^1/a_0^1|,$$
$$\beta = \cos\delta^- \equiv \cos(\phi_2^1 - \phi_0^1) \ .$$

Eq. (8a) allows to express q through K_z^z and β

$$q_{1,2} = \frac{\sqrt{2}\beta \pm D}{3K_z^z - 1}, \quad (9)$$

with

$$D = \sqrt{9K_z^z(1-K_z^z) + 2(\beta^2 - 1)} \ , \quad (10)$$

giving two solutions for spin-transfer coefficient $K^x_{x\,1,2}$ as a function of K_z^z and β. These solutions are related to each other as $K^x_{x\,1}(K_z^z, \beta) = K^x_{x\,2}(K_z^z, -\beta)$ leading to constraints for K_i^i

$$-\frac{1}{\sqrt{2}}\frac{3}{\sqrt{8+\beta^2}} \le K_x^x \le \frac{1}{\sqrt{2}}\frac{3}{\sqrt{8+\beta^2}},$$
$$\frac{3-\sqrt{1+8\beta^2}}{6} \le K_z^z \le \frac{3+\sqrt{1+8\beta^2}}{6} \ , \quad (11)$$

with $d = \sqrt{9 + 2\beta^2}$. When $\beta^2 \simeq 1$, they reduce to

$$-\frac{\sqrt{2}}{2} \lesssim K_x^x \lesssim \frac{\sqrt{2}}{2}, \qquad 0 \lesssim K_z^z \lesssim 1 \ . \quad (12)$$

Interestingly, the found expressions for the spin-transfer coefficients in Eqs. (4) in terms of the partial amplitudes a_L^J coincide with the spin-transfer coefficients in the $\vec{p}p \to \Sigma^+\Theta^+$ reaction with $T = 1$ and negative parity. At the same time, Eqs. (8) coincide with the corresponding predictions for the reaction $\vec{N}N \to Y\Theta^+$ with $T = 0$ and positive parity [10], but the physical meaning of the corresponding partial amplitudes is quite different, because of differences in the initial states and the production mechanism.

Expressions for the spin-transfer coefficient are model dependent. They depend on the ratios r and q and phases α and β. However, the dependence of one spin-transfer element expressed through another one is almost model independent and therefore is more attractive. As an example, in Fig. 1 we show dependence of K_x^x as a function of K_z^z for $\alpha, \beta = \pm 1$.

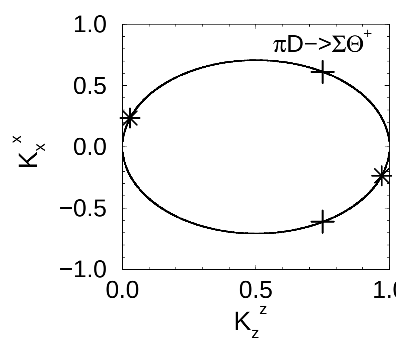

FIG. 1: The spin-transfer coefficient K_x^x in the reaction $\pi\vec{D} \to \vec{\Sigma}\Theta^+$ as a function of K_z^z for positive and negative Θ^+ parity at $\alpha, \beta = \pm 1$. The two solutions for $r = q = 1$ for positive and negative π_Θ are shown by crosses and stars, respectively.

In this limit the functions $K_x^x(K_z^z)$ do not depend on Θ^+ parity. But in spite of the similarity in $K_x^x(K_z^z)$ dependence one can find a strong difference between spin-transfer coefficients as a function of the partial amplitudes. Thus for example, if we accept a *democratic* choice of the partial amplitudes taking $r = q = 1$, then for the case of positive parity we find the solutions $K_z^z \simeq 0.75$ and $K_x^x \simeq \pm 0.61$. For negative parity we have two solutions $K_x^x \simeq -0.23$ at $K_z^z \simeq 0.97$ and $K_x^x \simeq 0.23$ at $K_z^z \simeq 0.03$, for positive and negative β, respectively. These solutions are located in the quite different places of the $K_x^x(K_z^z)$ plot, as is depicted in Fig. 1.

We have to note that the correlation $K_x^x(K_z^z)$ depends on the phases $\delta^\pm$ (or α, β), and is not model-independent. But at the same time, we expect that $|\alpha|$ and $|\beta|$ are close to one; thus this dependence is rather weak.

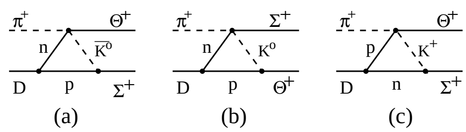

FIG. 2: Diagrammatic representation of the $\pi^+ D \to \Sigma^+\Theta^+$ reaction. For $\pi^- D \to \Sigma^-\Theta^+$ one has to exchange $\pi^+ \to \pi^-$, $\Sigma^+ \to \Sigma^-$ and $n, p \to p, n$.

Indeed, the dominant contribution to the production amplitude depicted in Fig. 2 comes from the imaginary parts of the corresponding diagrams obtained by cutting the loops as shown schematically in Fig. 3a, b and c (T_p). The non-pole (background) contribution (T_{bg}), shown in Fig. 3d, is much weaker [14]. This means that the phase differences $\phi_1^0 - \phi_1^1$ and $\phi_1^0 - \phi_1^2$ are close to $\epsilon \pm \frac{\pi}{2}$ or $\epsilon + 0$ (depending on convention). For positive π_Θ, the relative phase between the partial amplitudes a_L^J with the same orbital momentum, $L = 1$, and different total angular momenta $J = 0, 1$ are defined by the spin decomposition which does not give an additional imaginary phase between a_1^0 and a_1^1. Similarly, for negative π_Θ, the partial amplitudes a_L^1 with $L = 0, 2$ defined by the angular harmonic decomposition of the total amplitude have an orbital phase factor $i^L = \pm 1$ which does not provide additional imaginary phase between a_0^1 and a_2^1. Therefore, the corresponding relative phases $\delta^\pm$ in Eqs. (4) and (8) can be estimated as $|\cot\delta^\pm| \sim |T_p|/|T_{bg}|$ being close to 0 or π.

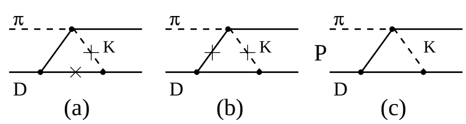

FIG. 3: Diagrammatic representation of cutting (shown by crosses) the loop diagrams (a) and (b); (c) corresponds to non-pole background contribution.

Fig. 4 shows the dependence of the spin-transfer coefficients K_x^x on K_z^z for two values of the relative phases, $\delta^\pm = 0^o$ and 30^o for positive (right panel) and negative (left panel) π_Θ. One can see a tiny modification of K_x^x when the phases $\delta^\pm$ become finite.

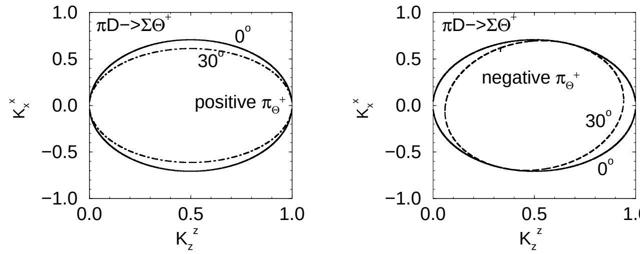

FIG. 4: The spin-transfer coefficient K_x^x $\pi D \to \Sigma\Theta^+$ as a function of K_z^z at two values of the relative phases $\delta^\pm = 0^o$, 30^0 for positive (left panel) and negative (right panel) π_Θ.

III. DEUTERON SPIN ANISOTROPY

Let us consider now the dependence of the probability of $\Sigma\Theta^+$ production as a function of the angle θ between deuteron spin polarization and the direction of the pion beam. For this aim we analyze the angular distribution $W(\cos\theta)$, normalized as

$$\int_{-1}^{1} W(\cos\theta)\, d\cos\theta = 1 \ . \qquad (13)$$

This distribution may be found by using the general form of the production amplitude in Eq. (3) and choosing the quantization axis along the deuteron spin. It has the universal form

$$W^\pm(\cos\theta) = \frac{3}{2(3+\mathcal{A}^\pm)} \left(1 + \mathcal{A}^\pm \cos^2\theta\right) \ , \qquad (14)$$

where $\mathcal{A}^\pm$ is the deuteron spin anisotropy, and the superscript $\pm$ indicates the Θ^+ parity. The anisotropies may be expressed in explicit form through the partial amplitudes

$$\mathcal{A}^+ = \frac{3r^2 - 2}{3r^2 + 2} \ , \qquad (15a)$$

$$\mathcal{A}^- = \frac{3(2\sqrt{2}\beta q - q^2)}{4 - 2\sqrt{2}\beta q + 5q^2} \ . \qquad (15b)$$

Using Eqs. (5) and(9), we find the following relations

$$\mathcal{A}^+ = \mathcal{A}^- = \frac{3K_z^z - 2}{2 - K_z^z} \ . \qquad (16)$$

One can see that the asymmetry $\mathcal{A}$ as a function of K_z^z does not depend on π_Θ. It is also important, that the shape of the spin anisotropy as a function of K_z^z does not depend on the phases $\delta^\pm$ and therefore is fully model independent. Again, in spite of the universality of the shape $\mathcal{A}(K_z^z)$ we find a strong difference in dependence

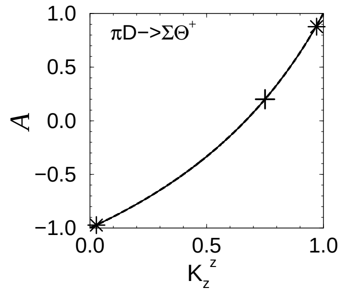

FIG. 5: The deuteron spin-transfer anisotropy $\mathcal{A}$ as a function of K_z^z. The solutions for $r = q = 1$ for positive and negative π_Θ are shown by cross and stars, respectively.

of the anisotropy on the partial amplitudes for different parities. As an example, in Fig. 5 we show solutions for positive and negative π_Θ at $r = q = 1$. One can see a strong difference in $\mathcal{A}$ for these different cases. For positive π_Θ, $\mathcal{A} \simeq 0.2$, whereas for negative π_Θ we have $\mathcal{A} \simeq \pm 1$, depending on phase of β. This strong difference may be studied experimentally.

IV. SUMMARY

In summary, we have analyzed two types of spin observables in the reaction $\pi\vec{D} \to \vec{\Sigma}\Theta^+$ near the threshold. One type of observables concerns the spin-transfer coefficients K_x^x and K_z^z. Another one is the spin anisotropy. We found a model independent correlation between the anisotropy and spin-transfer coefficient K_z^z. Each of these observables has their own dependence on the partial amplitude and may be calculated in a corresponding dynamical model which may be used for the determination of the Θ^+ parity.

For the practical implementation of the suggested measurements one needs a pion beam at fixed energy imping-ing a polarized deuteron target and a large phase space detector for the identifying $\Theta^+ \to NK$ and $\Sigma \to N\pi$ decay channels. The $\vec{\Sigma}^\pm$ polarization is measurable in the standard way, by analyzing the angular distribution in the decay channel. The available secondary pion beam delivered at the SIS/GSI Darmstadt and the spectrometers FOPI and HADES make such a measurement feasible. As a first step one can study the spin anisotropy which needs only a polarized deuteron target but does not require a measurement of the spin of outgoing particles.

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